# Design Experience with Nonlinear Tension Based Systems: Tents, Trusses and Tensegrity

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## Abstract

Experience with tensile membrane structures has lead the authors to a more general approach to structural design of more traditional systems. Design of structural systems intentionally eliciting non-linear large deflection behavior has achieved benefits. This has lead to the design of unique tension based, under-determinate and/or prestressed spatial systems.

The principles of designing with geometric stiffness are discussed, including prestress as a property of a structural system and deformation as a means of load response attenuation.

The authors present a wide variety of structural systems which benefit from geometric stiffness by design.

These attributes of tensile membrane structures can be developed and exploited in a wide variety of tensile based systems.

### Introduction

A structure responds to load on it by changes in internal stresses and by displacement of internal forces. This later response, geometric stiffness, is ignored in conventional structural theory with the presumption of "small" deflection. As this term is a product of both deflection and the internal force, it will only be inconsequential when the product is small in relation to changes in stress. The introduction of prestress in a conventionally stiff structural system can increase the internal forces sufficiently to make this term significant. Indeed Levy and Spillers have demonstrated the theorem:

"In the presence of prestress, geometric nonlinearities are of the same order of magnitude as linear elastic effects in structures." (Levy)

In tensile membrane structures the geometric stiffness is often greater than the elastic stiffness. It is common for tensile structures to have many non-extensional modes of deformation which are resisted solely by geometric stiffness. In general, elastic deformation contributes to geometric stiffness. It is this aspect of tensile structures which makes them different than other structures. They cannot be analyzed without due consideration of this nonlinear behavior.

Long experience with these systems has lead to a general perspective in structural design where nonlinear behavior is considered and indeed encouraged. Considerable advantage can be realized in tension based systems where design experience has demonstrated the significance of the Levy-Spillers Theorem. Moreover, member/element prestress has become a design parameter to be established as important as the elastic properties.

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## Tensile Membrane Structures

As design experience with tensile membranes increased, it became natural to exploit geometric stiffness. The initial means, at least in the authors' practice, was to allow greater movement and greater flexibility. Initially this was manifest in the design of tent structures in the elimination of redundant cables to allow greater system deformation. An example from 1973 was the conical tent designed by Geiger Berger for Great Adventure in New Jersey. In analyzing various design options optimal performance of the tensile membranes under wind loads occurred where the tent's mast was free to rotate. (Geiger, 1989) ( Berger).

In the author's practice there has been a steady trend to reduce the design prestress of large tent-form tensile structures. Initially it was deemed necessary to establish prestress in the membrane at nominally 1/10th of the tensile strength of the material. This was assumed to be a maximum permissible membrane stress under prestress and dead load. Prestress at this level was deemed necessary to achieve the maximum geometric stiffness possible. In anticlastic surface membranes this also stored the maximum strain energy in direction of curvature which would unload under a given load condition. All this was aimed at reducing deformations and to prevent flutter and other dynamic instabilities.

An example of this philosophy is the Haj Terminal at Jeddah International Airport. Designed in 1978 the membrane prestress for the Haj modules was 123 N/cm. With experience it was recognized that this level of prestress was not necessary. Structures such as the roof of Canada Harbour Place in Vancouver, B.C. (designed in 1984) were designed with a membrane prestress of 61 N/cm or half of that of the Haj Terminal.

**Figure 1**

The advantages in construction of lower prestress are obvious. Benefits are achieved in the installation and stressing of the structure as less prestress directly equates to less work. As the geometric stiffness is reduced, greater deformation is required for the structure to resist a given load. In membrane structures which consist of a continuum, this generally means a larger portion of the structure is engaged in resisting a given load distribution. A consequence is that a greater portion of the system is engaged in resisting local loads. The result is that the effects of load concentrations are attenuated in the system. Said in another way: local loads are resisted by local deformations rather than local changes in stress. With the caveat that the deformations remain within serviceable bounds, such structures are remarkably insensitive to load distribution. Deformations may in fact be quite large in contrast to either compression or flexure based structures. For example the maximum deflection of the Canada Harbour Place roof under non-uniform drifted snow is predicted to be nominally 2 m on a 71 m span. See Fig. 1.

**Figure 2**

More recent tensile structures such as the roof on the Ja-Yi Gymnasium in Ja-Yi, Taiwan, designed in 1995, are representative of structures which are intentionally allowed to deform. Design prestress in this structure is a relatively low 35 N/cm. Maximum deflections are 1.62 m from wind uplift on a 50 m span. See Fig. 2.

## Pleated Membranes

The necessity of adequate local curvature in tensile membranes is axiomatic. In the tent-form structures this generally means finding an anticlastic prestressed form with appropriately small radii of curvature. This is not always possible and other means of assuring suitable strength are necessary. Of course it is not the prestress curvature that is important but the curvature of the membrane in its deformed condition under load which dictates the membrane stress.

**Figure 3**

"Pleated" membrane structures present an interesting case in point. The roof of the Lindsay Park Sports Centre in Calgary, Alberta, (Fig 3 ) was completed in 1983. The roof membrane was employed in a manner that was a departure from previous tensile structures. The radii of the saddle surfaces which comprise the roof were too large for the strength of PTFE coated fiberglass. The design solution was to "pleat" the membrane by pulling it up out of the surface of the cablenet saddle with ridge cables at roughly 10 m centers. While this ensured regular reinforcing of the membrane with cables in the sagging direction, the radii of curvature of the membrane was still very large. However, each "pleat" has sufficient surface length from ridge to ridge or valley to valley, which combined with the relatively soft boundary support of cables, allows the membrane to develop significant local curvature under load. In this manner the membrane always spans in the same direction by reversing curvature with reversals in load. The softness of the cable boundaries of the "pleat" assure that the membrane never goes slack in this cycle. Consequently, the radii of curvature in the prestress form are of little use in predicting membrane stress under load.

The use of "pleated" tensile membrane was central to the realization of the first tensegrity type domes, the Geiger Cable Domes. (Tuchman) It has also found extensive use in a wide variety of structures in Japan. Well executed examples include the Akita Sky Dome, the Komatsu Dome and the Akita O-Dome. (Ishii)

The same approach was adopted in the preliminary design of a large radio telescope enclosure for the summit of Sera La Negra, Mexico. (Fig 4.) The enclosure concept developed by the Instituto Nacional Astrofisico Optical y Electronico was a co-rotating dome with a tensile membrane window covering the 50 m diameter telescope aperture. The only membrane suitably transparent to the radio spectrum is a fabric of woven PTFE yarn which has relatively low tensile strength. The primary radii of curvature of the anticlastic 58.4 m x 110 m "window" in the dome were much too large for the required membrane. The solution was to create a pleated membrane with valley cables in the hogging direction "pulling" the membrane down from a cablenet. Analysis demonstrated that the membrane stresses could be kept within permissible limits with this design.

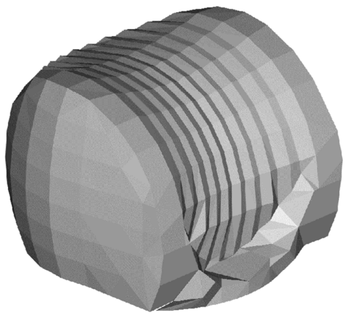


**Figure 4**

## Tensegrity

When Geiger first proposed the Cable Dome (Geiger), we supposed that it would be too reliant upon large deflection geometric stiffness in resisting non-uniform loads to be serviceable. To address this concern cross cables were included in each circumferential panels between the struts to stiffen the system. (ENR ) As the concept was developed these elements were found to be irrelevant and problematic. In fact the system was considerably less sensitive to load distribution without these elements. What was not appreciated initially was the effect of the tension hoops in attenuating localized load effects throughout the system. The geometric stiffness is relatively large in non-extensional modes of deformation as the tension in the hoops is large. The tension hoops resist out-of-plane deformation. Interestingly, by interconnecting all radials in the system, changes in hoop tension cause a response in the entire cable and strut network. The advantages of this in minimizing local effects of concentrated loads has been demonstrated. (Campbell, 1994)

**Figure 5**

Ironically, while the geometrically nonlinear behavior is often referred to as "large deflection" behavior, deflections need not be large as can be construed from the Levy -Spillers Theorem. The Cable Dome structures well illustrate this point. In the case of the Crown Coliseum roof the maximum deflection under large nonsymetric load conditions is nominally 230 mm on a span of 99.7 m. See Fig 5. (Gossen, 1998) This point has been missed by some who view the system from a linear elastic context. (Mikhailov).

**Figure 6**

Interestingly, most of the Cable Domes have little prestress, i.e. the member forces are small when the system is not subjected to load. The majority of the hoop tension in the self-weight condition is a consequence of the self weight. The attendant geometric stiffness applies to changes in load from the self-weight condition. The conventionally clad Crown Coliseum is stiffer than other membrane clad Cable Domes of similar size such as the roof on Taoyuan Arena, completed in 1994, (Fig 6) because of the greater mass of its rigid roof panels.

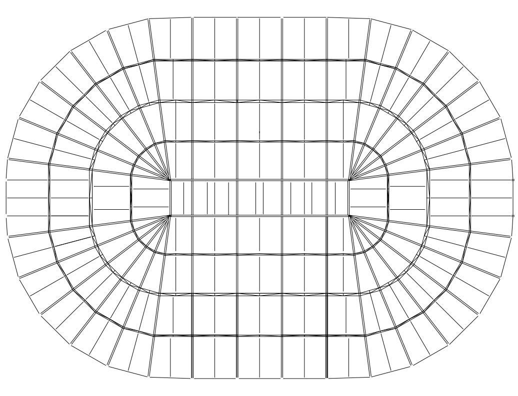
## Trusses

The concepts developed from years of experience in tensile structures have been successfully employed in more conventional spatial trusses. The prestressed spatial trusses of the North Charleston Coliseum, North Charleston, SC (Gossen, 1992) and the BI-LO Center Arena, Greenville, SC were designed with this approach.

The BI-LO Arena roof consists of a radial/circumferential steel truss network spanning 89 m x 124 m. See Fig. 9. The radial elements consist of Fink trusses. See Figs. 11 & 12. The bottom chord ties are steel cables which radiate from four hubs and a center tension "frame". See Fig. 10. These are all interconnected by a cable tension hoop. The entire network was prestressed by jacking the struts which connect the radial truss elements to the cable tie network. Construction photographs of the system showing the telescoping struts are shown in Figs. 7 and 8.

Prestress was established in the design such that the radial and circumferential top chords would have a prestress in the middle of their stress range for applied load. The cable arrangement of the bottom chord has a similar attenuating effect as the tension hoops of the Cable Dome. The structural system demonstrated the desired insensitively to load distribution, an advantageous attribute in arena roof structures which are subject to large concentrated loads from entertainment rigging.

As the prestress was determined solely on the basis of the benefits achieved under load, it was necessary to analyze the structure by systematic de-construction to determine the stressing sequence and procedure. This technique of determining stressing sequence was developed to address complex tensile structures. (Campbell, 1991) Prestress was verified by measurement of cable tensions at the completion of the jacking sequence.



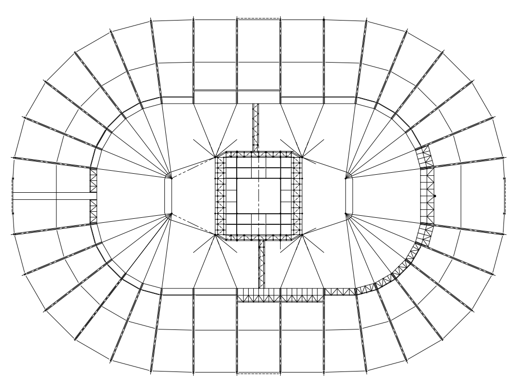


**Figure 9: Top Chord Plan**

**Figure 10: Bottom Chord Plan**

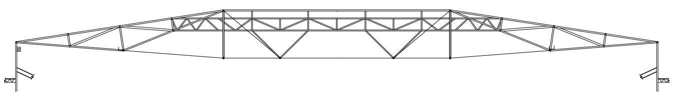


**Figure 11 : Longitudinal Section**

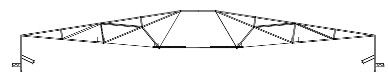


**Figure 12: Transverse Section**

## Closing Remarks

Intentionally developing structures to exploit nonlinear geometric stiffness raises some interesting questions which require further thought:

1. How should Load Resistance Factored Design methodology be applied to structures which exhibit geometric non-linear behavior? Super position of structural response is meaningless. Factoring prestress in a system changes an attribute of the system and effects its response to applied load. Understanding the structural condition caused by factored loads does not necessarily provide insight into service conditions. (Hamilton)

2. The dynamic behavior of systems with large geometric stiffness is difficult to address as the stiffness of the system varies with amplitude of deformation.

3. Deformations must be understood throughout the range of service conditions and addressed in design.

4. Prestress must be reliably developed and verified if it is a critical aspect of the structural design.

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